Math for AI

Syllabus

- Linear algebra
- Calculus + optimization methods
- Probability theory + statistics

AI Academy

Linear Algebra

AI Academy

01 Introduction to vectors, matrices

1. Vectors and linear combinations
2. Lengths and dot products
3. Matrices, definition, matrix operations

02 Linear equations and Gaussian elimination

1. Vectors and linear equations
2. Gaussian elimination
3. Elimination using matrices

03 Matrix operations and decomposition

1. Rules for matrix operations
2. Inverse matrices, conditions for existence, Gauss-Jordan elimination, Determinant
3. LU decomposition; Transposes, symmetric, permutation matrices

04 Vector spaces and subspaces

1. Vector spaces, subspaces
2. The nullspace of A: Solving Ax = 0
3. The complete solution to Ax = b

05 Independence, basis and dimension, orthogonality

1. Linear independence, basis, dimension of vector spaces
2. Four fundamental subspaces (column, null, row, left null) and their dimensions
3. Orthogonal subspaces, orthogonal complements

06 Orthogonality (cont’d)

1. Orthogonal bases, Gramm-Schmidt process
2. Projection of vectors, orthogonal projections
3. Least squares approximation, fitting linear models

07 Eigenvalues and eigenvectors

1. Eigenvalues, eigenvectors, characteristic equation
2. Diagonalization, conditions for diagonalization
3. Iterative estimates for eigenvalues /eigenvectors

08 Linear transformations

1. Definitions and properties of linear transformations
2. Representation of linear transformations as matrices
3. SVD, computation and applications

09 Final exam and projects

1. Review session
2. Comprehensive assessment covering all topics (final exam)
3. Presenting final projects that apply multiple linear algebra concepts

Calculus + optimization methods

AI Academy

01 Fundamentals of calculus for AI/ML

1. Introduction to Limits, definition, limit laws
2. Continuity, types of discontinuities
3. Limits at Infinity and Infinite Limits

02 Differential calculus and its applications

1. Introduction to derivatives, definition, rules of differentiation
2. Applications of derivatives, tangent lines, rates of change
3. Second and higher-order derivatives, Taylor expansion

03 Integral calculus and its applications

1. Introduction to integrals, definition, fundamental theorem of calculus
2. Applications of integrals, areas, volumes
3. Techniques of integration (integration by parts, substitution)

04 Multivariable calculus for AI/ML

1. Partial Derivatives, chain rule
2. Gradient vectors, Hessian matrices, linear and quadratic approximations
3. Double integrals and applications

05 Optimization techniques in AI/ML (part1)

1. Gradient Descent Variants - stochastic, mini-batch
2. Gradient Descent with Momentum, Adaptive Momentum (Adam); Newton's method
3. BFGS

06 Optimization techniques in AI/ML (part2)

1. Conjugate gradient
2. Trust-region methods

07 Optimization techniques in AI/ML (part3)

1. Constrained optimization, Lagrange multipliers
2. KKT conditions

08 Advanced AI/ML applications

1. Case studies in AI/ML
2. Integrating techniques in AI/ML, combining calculus and optimization
3. Review and preparation for the final project

Probability theory + statistics

AI Academy

01 Introduction to probability theory

1 Fundamentals of probability

- Basics of probability, event spaces

2 Conditional probability, independence, law of total probability, chain rule (product rule)

3 Random variables

- Discrete random variables, probability mass function (PMF)

02 Intro (cont'd)

1 Random variables (cont'd)

- Continuous random variables, probability density function (PDF)

2 Probability distributions

- common distributions (Bernoulli, beta, binomial, exponential, gamma, normal, Poisson)

3 Probability distributions (cont'd)

- bivariate, marginal/conditional distributions, independent random variables

03 Expectation, variance, and moments

1 Expectation and variance

- Definition and properties of expectation and variance

2 Moments and moment generating functions

- Higher-order moments, moment generating functions

3 Covariance and correlation

- Covariance, correlation, and their properties

04 Bayesian probability

1 Bayes theorem

- Bayes theorem, applications, and examples

2 Bayesian inference

- Bayesian inference, prior and posterior distributions

3 Bayesian networks

- Intro to bayesian networks

05 Hypothesis testing and confidence intervals

1 Hypothesis testing basic

- Null and alternative hypotheses, types of errors

2 Confidence intervals

- Constructing and interpreting confidence intervals

3 Advanced hypothesis testing

- ANOVA, chi-square tests

06 Statistical inference and estimation

1 Point estimation

- Methods of point estimation, properties of estimators

2 Interval estimation

- Constructing and interpreting interval estimates

3 Maximum likelihood estimation (MLE)

- Principles of MLE, applications

07 Markov chain and sampling methods

1. Markov chain

2. Standard distributions
   - Multinoulli, Gaussian

3. Importance sampling, MCMC, Gibbs sampling

08 Review and final project

1. Review of the content
2. Final project